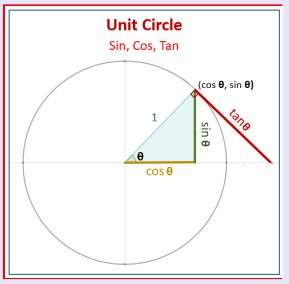
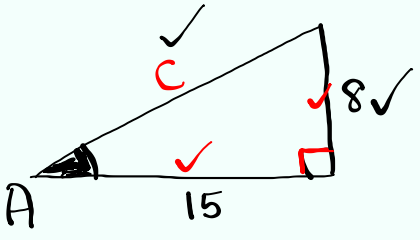
Trigonometry

Lecture 13



Given



Class Quiz 3

1) Find the hypotenuse.

$c^2 = 8^2 + 15^2$

$= 64 + 225$

$= 289$

$c = 17$

2) Find $\sin A = \frac{8}{17}$

Simplify

$$\sin x \cdot \cot x \cdot \sec x$$

$$= \sin x \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}$$

$$= 1$$

Simplify

$$(\sin x + \cos x)^2 + (\sin x - \cos x)^2$$

$$= \sin^2 x + 2\sin x \cos x + \cos^2 x + \sin^2 x - 2\sin x \cos x + \cos^2 x$$

$$= 1 + 1$$

$$= \boxed{2}$$

Recall

$(A+B)^2 = A^2 + 2AB + B^2$

$(A-B)^2 = A^2 - 2AB + B^2$

Verify $\sec x - \csc x = \dfrac{\sin x - \cos x}{\sin x \cos x}$ ✓

$$\frac{\sin x - \cos x}{\sin x \cos x} = \frac{\sin x}{\sin x \cos x} - \frac{\cos x}{\sin x \cos x}$$

$$= \frac{1}{\cos x} - \frac{1}{\sin x}$$

$$= \boxed{\sec x - \csc x}$$

Verify

$$(\sec x - \tan x)^2 = \frac{1-\sin x}{1+\sin x}$$ ✓

$$(\sec x - \tan x)^2 = \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2$$

$$= \left(\frac{1-\sin x}{\cos x}\right)^2 = \frac{(1-\sin x)^2}{\cos^2 x}$$

Recall

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

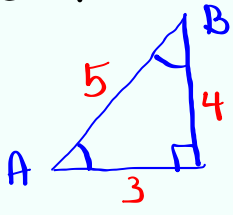
$A^2 - B^2 = (A+B)(A-B)$

$$= \frac{(1-\sin x)^2}{1-\sin^2 x}$$

$$= \frac{(1-\sin x)^2}{(1+\sin x)(1-\sin x)}$$

$$= \boxed{\frac{1-\sin x}{1+\sin x}}$$

Draw the famous 3-4-5 right triangle.



Find the measure of all three angles in nearest degree. $\angle C = 90^\circ$

$A + B = 90^\circ$

$53^\circ + 37^\circ = 90^\circ$ ✓

$\sin A = \frac{4}{5}$ $\quad \sin A = .8$

$A = \sin^{-1}(.8) \approx 53^\circ$

$\cos B = \frac{4}{5}$ $\quad \cos B = .8$

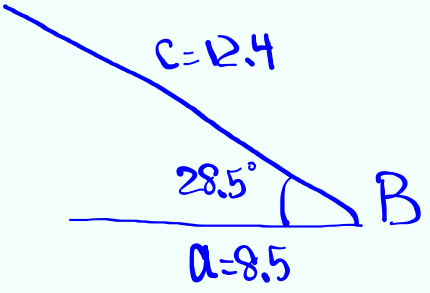
$B = \cos^{-1}(.8) \approx 37^\circ$

Find area of triangle ABC with

$a = 8.5$ in., $c = 12.4$ in., and $B = 28.5°$.

Draw such triangle.

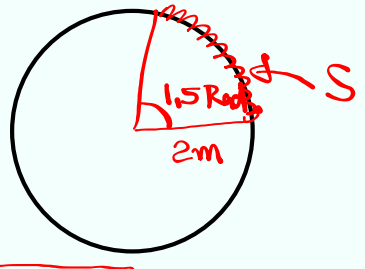
SAS



Area $= \frac{1}{2} ac \sin B$

$= \frac{1}{2}(8.5)(12.4)\sin 28.5° \approx \boxed{25.1 \text{ in}^2}$

Draw a Sector with Central angle of 1.5 radians and radius of 2 m.



1) Find its area

$A = \frac{1}{2} r^2\theta = \frac{1}{2} \cdot 2^2 \cdot (1.5) = \boxed{3 \text{ m}^2}$

2) Find its arc length. $S = r\theta$

$= 2(1.5) = \boxed{3\text{m}}$

The road to the top of a hill has a 6.5° angle with the flat land.

From the bottom of hill to top is 1.5 miles.

How tall is the hill?

$\sin 6.5^\circ = \frac{h}{1.5}$ $\quad h = 1.5 \sin 6.5^\circ$

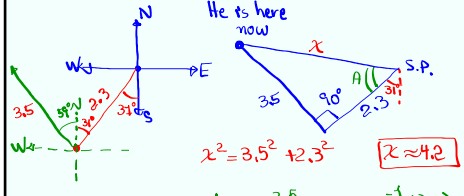
$\approx .2$ miles

Alejandro walks 2.3 miles in the direction of S 31° W.

Then he turns 90° and walks 3.5 miles in the direction of N 59° W.

1) How far is he from the starting point?

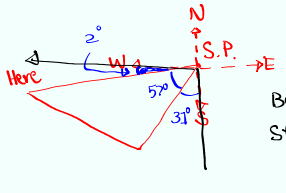
2) What is his bearing from his starting point?



$x^2 = 3.5^2 + 2.3^2$ $\quad x \approx 4.2$

$\tan A = \frac{3.5}{2.3}$ $\quad A = \tan^{-1}\left(\frac{3.5}{2.3}\right)$

$= 57^\circ$

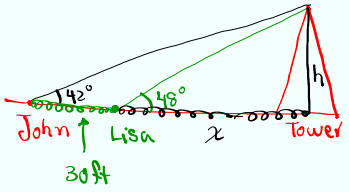


Bearing from Starting Point

S 88° W

John's angle of elevation to the top of a tower is 42°.

Lisa is 30 ft closer to the tower, her angle of elevation to the top of the tower is 48°. How far is John from the tower?



$\tan 48° = \frac{h}{x}$

$h = x \tan 48°$

$\tan 42° = \frac{h}{x+30}$

$h = (x+30) \cdot \tan 42°$

$(x+30)\tan 42° = x \tan 48°$

$x \tan 42° + 30 \tan 42° = x \tan 48°$

$30 \tan 42° = x \tan 48° - x \tan 42°$

$30 \tan 42° = x[\tan 48° - \tan 42°]$

$x = \frac{30 \tan 42°}{\tan 48° - \tan 42°} \approx 129$ ft

John is 129 ft from tower.